量子準位ダイナミックスの確率過程理論(筑波大学開学20周年記念第2回『非平衡系の統計物理-現状と展望』シンポジウム,研究会報告)

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量子準位ダイナミックスの確率過程理論

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ここにいう'準位ダイナミックス'(level dynamics) とは、多数の複雑量子準位が断熱パラメーターに対して変化する chaotic な様相をそのパラメーターに関する微分方程式の解としてとらえる手段を意味していて 1985、86 年の Yukawa の論文[1]に始まる[注]。ついで、Haake[2]のグループが主としてユニタリ行列系を主な対称とするこの手段の開発を試みた。この手段の興味ある応用に'準位の曲率分布の問題'がある。それは、各レベルのパラメーターに関する 2 次微分 (曲率) を統計的に扱ってその確率分布としてみる、というもので 1990 年 Gaspard ら[3]が論じた。その一つの重要な結論は、曲率を K とするとき分布 $P(K)$ の $K \to \infty$ での振舞が $K^{-(\nu+2)}$ ($\nu = 1$ GOE、$\nu = 2$ GUE、$\nu = 4$ GSE) という普遍則に従う、というものであって、われわれが幾つかの例で確認したものである。[4]

この報告では、準位ダイナミックスを確率過程として扱うことにより分布 $P(K)$ が統一的に見られることを示す。それは、準位ダイナミックスに久保のブラウン運動論[5]を適用するというアイデアである。以下、この報告記事の構成について説明する。

Part I.　準位ダイナミックスに関する Yukawa 形式の要約
　本報告に必要な最小限にとどめ、その特定の 2 準位の運動をランジュバン方程式化することについて説明を加える。

Part II.　**Nonuniversality in Curvature Distributions of Quantum Levels**
　われわれの得た解析的ならびに数値的結果の速報で、1993 年 8 月末に行われた福井国際シンポジウム 'Quantum Chemistry and Technology in the Mesoscopic Level' において発表したもの。

Appendix　上述の英文原稿において出発点となる $P(K)$ の積分表式 (3) の補足説明

文献

1) T. Yukawa: Phys. Rev. Lett. **54** (1985) 1883; Phys. Lett. **A116** (1986) 227.
2) F. Haake: Quantum Signature of Chaos (Springer Verlag, Berlin 1991), Chap. 6.
3) P. Gaspard, S.A. Rice, H.J. Mikeska and K. Nakamura: Phys. Rev. **A42** (1990) 4015.
4) T. Takami and H. Hasegawa: Phys. Rev. Lett. **68** (1992) 419.
5) 岩波講座 現代物理学の基礎 6「統計物理学」第 5、6 章。

[注] より詳しくいえば、ランダム行列理論の結果を動力学的に再現しようと試みた Pechukas (1983)、更に 1960 年代ランダム行列理論を展開した Dyson がその中の一つとして準位群のブラウン運動論を論じたことにその始まりを認めることもできる。それで、これを一括して Dyson–Pechukas–Yukawa 像と呼ぶ人もいる。30 年前に遡る Dyson の理論を再構成することはこの研究の先の目標である。

Part I.　準位ダイナミックスとその確率過程化

$N \times N$ エルミート行列 $H = H_0 + \lambda V$ の固有値問題

$$(H_0 + \lambda V)u(\lambda) = x(\lambda)u(\lambda)$$

は、N-固有値 $\{x_n(\lambda)\}$ を含む $2N + \nu N(N-1)/2$ 変数の連立常微分方程式で置き換えられることが知られている。[1)]

$$\frac{dx_n}{d\lambda} = p_n, \qquad \frac{dp_n}{d\lambda} = 2\sum_{m(\neq n)} \frac{||f_{mn}||^2}{(x_n - x_m)^3} \tag{1}$$

$$\frac{df_{mn}}{d\lambda} = \sum_{l(\neq m,n)} f_{ml} f_{ln} \left[\frac{1}{(x_m - x_l)^2} - \frac{1}{(x_n - x_l)^2}\right] \tag{2}$$

$$p_n \equiv (u_n, Vu_n), \qquad f_{mn} \equiv (x_m - x_n)(u_m, Vu_n), \tag{3}$$

$$||f_{mn}||^2 = \sum_{a=0}^{\nu-1} |f_{mn}^a|^2 \qquad \begin{pmatrix} \nu = 1 & V\text{-real symmetric} \\ 2 & V\text{-complex symmetric} \\ 4 & \text{quaternion real} \end{pmatrix} \tag{4}$$

さらに、この力学系は変数間のポアソン括弧 (P.B.) を適切に与えることにより次に与えるハミルトン関数 $\mathcal{H}$ のハミルトン力学系 $\frac{d}{d\lambda}y_\alpha = \{y_\alpha, \mathcal{H}\}$ であることがわかっている:

$$\mathcal{H} = \sum_{n=1}^{N} \frac{1}{2}p_n^2 + \sum_{m<n} \frac{||f_m n||^2}{(x_m - x_n)^2} \tag{5}$$

$\{x_m, p_n\} = \delta_{mn}$、$\{x_m, x_n\} = \{p_m, p_n\} = \{x_n, f_{ij}\} = \{p_n, f_{ij}\} = 0$

$$\{f_{ij}, f_{kl}\} = -\sum_{(r,s)} C_{ij,kl}^{rs} f_{rs} \tag{6}$$

($C_{ij,kl}^{rs}$ は定数 — O、U、Sp のリー構造定数と呼ばれ、具体形は文献 3) 参照) したがって、N-レベルはあたかも逆 2 乗べき対ポテンシャルで互いに反発する 1 次元 N-粒子相互作用系とみることができる (g-CM 系)。Gaspard らの曲率分布理論も曲率 ($\frac{d^2}{d\lambda^2}x_n$) の統計処理をハミルトニアン (5) の正当的統計力学に委ねるものだった (対ポテンシャルの強さ $||f_{mn}||^2$ は定数ではなくて一つの力学変数として扱わねばならないことに注意 — f は N 次元回転の角運動量に相当)。ここでは、(多体問題の難しさを避けて)2 準位のみの運動に着目し、これに対する他の準位からの作用を確率化して扱うというブラウン運動論を用いて曲率分布を求める。それは、文献 (5) に述べられている考え方が自然にあてはまる好例ということができる。そして、いわゆる「運動による尖鋭化」という現象の取扱いとその結果が、まさしくわれわれが求めていた‘曲率分布の nonuniversality’ を明快に説明するものであることがわかったといえる。

運動方程式のランジュバン方程式化

方程式 (1) における特定準位対、例えば $n = 1, 2$ だけを取り出すと

$$\frac{dx_1}{d\lambda} = p_1, \quad \frac{dp_1}{d\lambda} = 2\frac{||f_{12}||^2}{(x_1 - x_2)^3} + R_1$$
$$\frac{dx_2}{d\lambda} = p_2, \quad \frac{dp_2}{d\lambda} = 2\frac{||f_{12}||^2}{(x_2 - x_1)^3} + R_2$$

となる。すなわち $\frac{dp_i}{d\lambda}$ の右辺で (1,2) 間の力だけはそのまま残し、他の項はそれぞれ R_i としてこれをランダム力 (Langevin force) とみなす。そのためにはそれに見合った減衰項を付加しなければならない。それがどのようなものでなければならないかについて文献 5) に詳細な議論がある (いわゆる‘おくれのある減衰’ と第 2 揺動散逸定理)。これらを考慮してたてたのが Part II の方程式 (17) である。

Part II.

Nonuniversality in Curvature Distributions of Quantum Levels

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We are motivated by the recent theoretical analyses of Zakrzewski and Delande (Z–D) on curvature distribution of chaotic quantum levels to construct a unified theory which is capable of predicting the nonuniversal feature of the distribution $P(K)$: while $P(K)$ for large values of the curvature K of an irregular quantum level, embedded in a level diagram moving as a parameter λ (i.e. $K = \frac{d^2E}{d\lambda^2}$), has the universal power law $P(K) \sim K^{-(\nu+2)}$, $P(K)$ for $K \approx 0$ is different for different individual systems with variety of sharpened peaks, as we have demonstrated previously. We utilize the stochastic frequency modulation theory of Kubo who formulated the motional narrowing of resonant line shape in the context of Gaussian stochastic processes. Thus, a satisfactory scheme of interpolating the two limiting forms of $P(K)$ proposed by Z–D has been achieved.

1 Introduction

Our previous numerical investigation of the curvature distributions of complex quantum levels moving according to a change of an adiabatic parameter (Takami and Hasegawa[1]) revealed that, although the distribution (density function) $P_{\text{curv}}(K)$ for large K values behaves like $K^{-(\nu+2)}$, $\nu = 1$ for GOE and $\nu = 2$ for GUE, in agreement with the theoretical prediction of universality due to a level–dynamical formulation by Gaspard et al[2], its peak behavior around $K = 0$ differs greatly from one sample to another chosen in the numerical tests (see Fig. 1). This feature about nonuniversality has been one of the major clarifying points of the recent theoretical analyses on the parametric motion of chaotic quantum levels contained in two papers by Zakrzewski and Delande[3] (in abbreviation Z–D hereafter) and Zakrzewski et al[4]. See also the preceding review in this volume[5].

In this report we discuss a probabilistic formulation of the curvature distribution of levels for a fully chaotic quantum system, where the origin of the two formulas for $P_{\text{curv}}(K)$ proposed in Z–D will be clarified in terms of the 'relative correlation length in the parameter' of

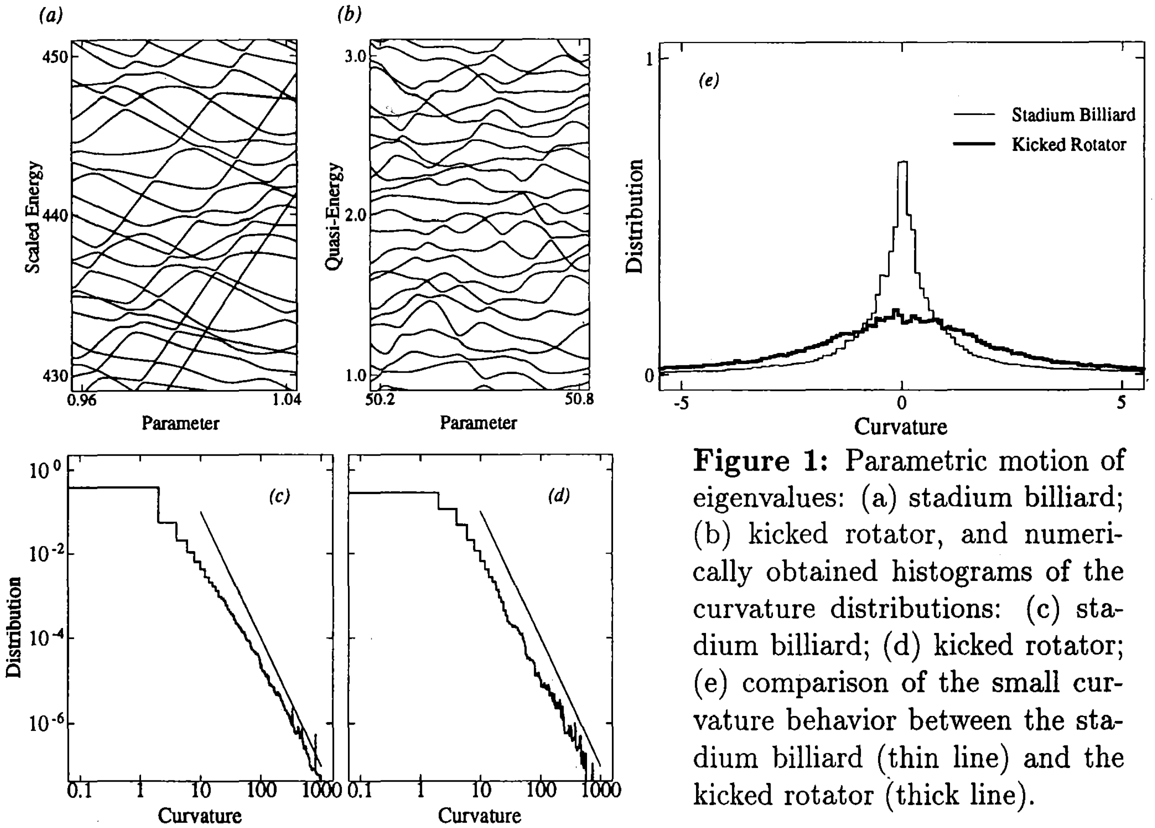

Figure 1: Parametric motion of eigenvalues: (a) stadium billiard; (b) kicked rotator, and numerically obtained histograms of the curvature distributions: (c) stadium billiard; (d) kicked rotator; (e) comparison of the small curvature behavior between the stadium billiard (thin line) and the kicked rotator (thick line).

random influence acting on the pair of moving levels: The first formula (eq. (4.15) in Z–D, constructed as a two-level statistical model of the level dynamics) reads

$$P_{\rm curv}(K) = C_\nu K^{\frac{\nu-2}{2}} \mathcal{D}_{-\frac{3}{2}\nu-1}(B_\nu K) \tag{1}$$

with normalization and scaling constants C_ν and B_ν, and in terms of the parabolic–cylindrical related function

$$\mathcal{D}_{-p}(z) \equiv \frac{1}{\Gamma(p)} \int_0^\infty e^{-\frac{t^2}{2}-zt} t^{p-1} dt.$$

It corresponds to the shortest (zero) limits of the correlation length, implying that the level dynamics of the pair is taking place as if it were unaffected by any other levels. For GOE ($\nu = 1$), the formula yields a sharpened peak of $P_{\rm curv}(K)$ at $K = 0$.

On the other hand, the second formula (eq. (3.27) in Z–D, proposed just intuitively) which reads

$$P_{\rm curv}(K) = N_\nu (1 + B'^2_\nu K^2)^{-\frac{\nu+2}{2}} \tag{2}$$

with normalization N_ν, and another scaling constant B'_ν, is a consequence of the Gaussian stochastic assumption on the influence of all other levels than the pair, in the case of very long correlation length. Note also that the Wigner type function is assumed for the spacing distribution in both (1) and (2) (see below).

Our theoretical basis to deduce the above results is Kubo's stochastic frequency modulation theory[6] plus the so-called generalized Langevin method[6]: His concept of 'motional narrowing' is incorporated in the short-correlation formula (1). Numerical interpolation between the two limiting situations (1) and (2) will be exhibited.

2 Probabilistic Formulation

First, we point out that the curvature distribution $P_{\text{curv}}(K)$ can be expressed reasonably as an integral transform of the spacing distribution for an adjacent pair of levels $P_{\text{sp}}(s)$ such that

$$P_{\text{curv}}(K) = \int_{-\infty}^{\infty} F(K,s) P_{\text{sp}}(s) ds. \tag{3}$$

Here, we may assume the symmetry property

$$P_{\text{curv}}(-K) = P_{\text{curv}}(K), \quad P_{\text{sp}}(-s) = P_{\text{sp}}(s), \quad \text{and hence} \quad F(-K, \pm s) = F(K,s). \tag{4}$$

The probabilistic meaning assigned to the kernel function $F(K,s)$ must be that $F(K,s)$ represents the conditional curvature distribution associated to any pair of levels under the condition that two levels x_1 and x_2 arbitrarily chosen have the spacing value s i.e.

$$x_2(\lambda) - x_1(\lambda) = s, \tag{5}$$

where the curvature K associated to (x_1, x_2) is defined by

$$K \equiv \frac{d^2}{d\lambda^2} \frac{1}{2}(x_2(\lambda) - x_1(\lambda)). \tag{6}$$

We can show the correctness of the expression (3) with the additional assignment (5) and (6) along the line of statistical mechanical formulation by Gaspard et al (unrestricted to the tail part, however) [2], which should be presented elsewhere.

Our key tool for presenting the result stated in Section 1 is to investigate the characteristic function (i.e. the Fourier transform) of the distribution/conditional distribution, namely

$$\Phi(t) \equiv \int_{-\infty}^{\infty} P_{\text{curv}}(K)\, e^{iKt} dK = 2\int_0^{\infty} \Phi(t,s) P_{\text{sp}}(s) ds \tag{7}$$

$$\Phi(t,s) \equiv \int_{-\infty}^{\infty} F(K,s)\, e^{iKt} dK. \tag{8}$$

Then, the most important finding of Gaspard et al[2], namely the universal tail behavior of the curvature distribution $P_{\text{curv}}(K)$ can be transcribed by some simple scaling properties of $F(K,s)$ and its characteristic function $\Phi(t,s)$ as follows: Suppose that the spacing distribution $P_{\text{sp}}(s)$ assumes the power form $A_\nu s^\nu + O(s^{\nu+1})$ for small s values. The scaling property for F,

$$F(K,s) = |K|^{-1} f(Ks) \quad \text{such that} \quad \int_0^{\infty} f(k) k^\mu dk < \infty, \quad \mu \le -1, \tag{9}$$

or, that for Φ

$$\Phi(t,s) = \phi(\frac{t}{s}) \quad \text{such that} \quad \int_0^\infty |\frac{d^{\mu+1}}{dt^{\mu+1}}\phi(t)|dt < \infty \tag{10}$$

assures the universal tail behavior of $P_{\text{curv}}(K)$ as

$$P_{\text{curv}}(K) \overset{|K|\to\infty}{\longrightarrow} \bar{A}_\nu K^{-(\nu+2)}. \tag{11}$$

The two single–variable functions f and ϕ are related through

$$\phi(t) = \int_{-\infty}^{\infty} f(k)\, e^{ikt}\, \frac{dk}{|k|}, \quad f(k) = \frac{|k|}{2\pi}\int_{-\infty}^{\infty} \phi(t)\, e^{-ikt} dt. \tag{12}$$

At this moment, we must admit our ignorance about a deeper meaning of the scaling property (9) or (10). But, assuming its validity, we specialize to two typical examples which we designate as

(A) Case of the fast modulation ('motional narrowing' limit),

(B) Case of the slow modulation (Gaussian limit).

A. the fast modulation:

$$\phi(t) = \text{Re}\,(1-it)^{-\frac{\nu}{2}}, \quad f(k) = \frac{1}{2\Gamma(\frac{\nu}{2})}|k|^{\frac{\nu}{2}}\, e^{-|k|}. \tag{13}$$

Note that by means of a Laplace transform and its inversion

$$\begin{aligned}
\int_0^\infty \frac{1}{\Gamma(\frac{\nu}{2})} e^{-k(1+z)} k^{\frac{\nu}{2}-1} dk &= (1+z)^{-\frac{\nu}{2}} \\
\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \frac{e^{zk}}{(1+z)^{\frac{\nu}{2}}} dz &= \begin{cases} \frac{1}{\Gamma(\frac{\nu}{2})} k^{\frac{\nu}{2}-1} e^{-k} & \text{for Re } k > 0 \\ 0 & \text{for Re } k > 0 \end{cases}
\end{aligned}$$

B. the slow modulation:

$$\phi(t) = e^{-\frac{1}{2}t^2}, \quad f(k) = \frac{|k|}{\sqrt{2\pi}}\, e^{-\frac{1}{2}k^2}. \tag{14}$$

It is now clear how these two prototypes lead to the Z–D formulas (1) and (2) by adopting the Wigner type spacing distribution function

$$P_{\text{sp}}(s) = A_\nu s^\nu e^{-s^2/2\sigma_\nu^2}, \tag{15}$$

and by choosing the normalization and scaling constants adequately: It is immediate to obtain the formulas (1) and (2) by inserting (13) and (14), respectively, into the integral transform (3) of the Wigner function (15), where the scaling property (9) plays the decisive role.

3 Generalized (friction–retarded) Langevin Equation

Kubo[6)] discussed a retardation effect on the friction of the Ornstein–Uhlenbeck Brownian motion by writing its Langevin equation as

$$\frac{du}{dt} = -\int_{-\infty}^{\infty} \gamma(t-t')\ u(t')dt' + \frac{1}{m}R(t).$$

The necessity of such a generalization as expressed in the above form stems from the non-white nature of the random force $R(t)$ (i.e. $R(t)$ is not δ–correlated, or its power spectrum nonconstant). Thus it modifies the ordinary fluctuation–dissipation relation between the damping constant and the strength of the white noise such that

$$\int_0^{\infty} \gamma(\tau)\ e^{-i\omega\tau} d\tau = \frac{1}{mkT}\int_0^{\infty} < R(t)R(t+\tau) > e^{-i\omega\tau} d\tau. \tag{16}$$

Kubo called this the *fluctuation–dissipation theorem of the second kind.*

In order to apply this theory to the present problem of describing the variant correlation length, we first write down a set of two–level equation of motion in accordance with the level dynamics[7)] supplemented by a friction term and a random force:

$$\begin{aligned} \frac{dx_i}{dt} &= p_i \qquad i = 1,2 \\ \frac{dp_i}{dt} &= -\frac{\partial V}{\partial x_i} - \int_{-\infty}^{t} \gamma(t-t')\ p_i(t')dt' + R_i(t) \end{aligned} \tag{17}$$

where $V(x_1, x_2) = \dfrac{|L_{12}|^2}{(x_1 - x_2)^2}$ (see [2)]) and $R_i(t)$ $(i = 1,2)$ is assumed to arise from the gradient of the potential $\sum_{n\neq i} V(x_i, x_n)$ $(i = 1,2)$.

Then, let us take the center–of–mass coordinate system:

$$\begin{aligned} X &= \tfrac{1}{2}(x_1 + x_2), \qquad & P &= p_1 + p_2 \\ x &= x_2 - x_1, \qquad & p &= \tfrac{1}{2}(p_2 - p_1) \end{aligned} \tag{18}$$

This enables us to separate eq. (17) into two sets, and the one for the relative coordinates (x, p) is written as

$$\frac{dx}{dt} = 2p, \quad \frac{dp}{dt} = -\frac{\partial V}{\partial x} - \int_{-\infty}^{t} \gamma(t-t')\ p(t')dt' + r(t) \tag{19}$$

with the fluctuation–dissipation relation (of the 2nd kind) given by

$$\int_0^{\infty} \gamma(\tau)\ e^{-i\omega\tau} d\tau = 2\beta \int_0^{\infty} < r(t)\ r(t+\tau) > e^{-i\omega\tau} d\tau. \tag{20}$$

Here, β denotes the inverse temperature of the equilibrium surrounding the pair of levels which should be defined in the starting level dynamics, and the factor 2 represents the inverse reduced mass (note that each level has a mass unity in our level dynamics). The residual force r in (19), given by $\frac{1}{2}(R_2 - R_1)$, arises certainly because the pair of levels 1 and 2 interact with the other levels, $n \neq 1, 2$, for which the orthogonality $< R_i(t)R_j(t') > = 0$, $i \neq j$, is assumed. Then, the F–D relation (20) recovers its original form

$$\int_0^\infty \gamma(\tau)\ e^{-i\omega\tau} d\tau = \beta \int_0^\infty < R_1(t)R_1(t+\tau) > e^{-i\omega\tau} d\tau.$$

We are now ready to compute the characteristic function of the conditional curvature distribution and hence, with the aid of $P_{\mathrm{sp}}(s)$ in (15), the desired function $P_{\mathrm{curv}}(K)$ with a variant correlation length.

4 Stochastic Modulation Theory for Curvatures

Before going, we argue about what physical meaning should be assigned to the 'time' t in the Langevin equations (17) and (19). In the starting level dynamics, e.g. in Yukawa's formulation[8),7)], one deals with the eigenvalue problem of a Hamiltonian matrix

$$H(\lambda) = H_0 + \lambda H_1 \tag{21}$$

to ask a change of its energy eigenvalues with respect to λ. Here, the parameter λ is dimensionless as far as H_0 and H_1 have the same (energy) dimension. Change the parameter λ to t in the same expression but now H_1 being regarded as a dimensionless quantity:

$$H(t) = H_0 + tH_1.$$

Then, the parameter t now represents an energy variable. Accordingly, in eqs. (17) and (19), the momenta p's are dimensionless and their time derivatives are of the dimension [energy^{-1}]. At the same time, the choice of the dimensionless perturbing matrix H_1 implies that one now has a dimensionless Hamiltonian function for the level dynamics, which is convenient.

It is now possible to identify the curvature K for the pair of levels 1 and 2 with the right–hand side of the Langevin equation (19):

$$\frac{dp}{dt} = \frac{|L|^2}{x^3} - \int_{-\infty}^t \gamma(t-t')\ p(t')dt' + r(t) = K(t) \tag{22}$$

This is a fluctuating quantity against energy with its dimension [energy^{-1}]. It is consistent with the starting definition of the curvature (6) apart from the dimensional understanding.

Our task is to calculate the characteristic function $\Phi(t,s)$ of the curvature distribution under the condition specified by (5) such that $\Phi(t,s) = < e^{iKt}|x = s >$. Here, the variable t is the one conjugate to the curvature K in defining the characteristic function i.e. Fourier transform of $P_{\mathrm{curv}}(K,s)$, but now we assert that this variable t is identical to the time variable of the Langevin eq. (19). The reason for this identification can be seen from Kubo's another context of the Brownian motion theory for line shapes, namely the 'random frequency modulation theory'[6)]: He discussed a practical method to compute the

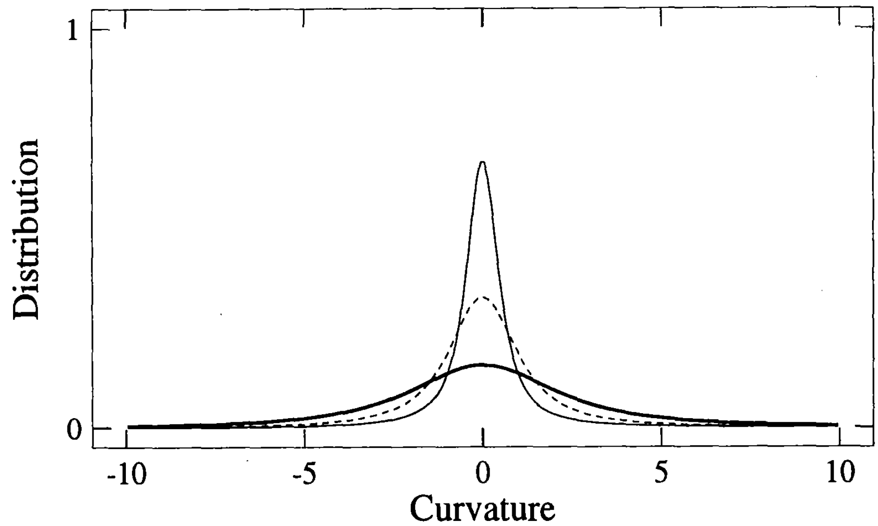

Figure 2: Curvature distribution for GOE statistics obtained from the intermediate characteristic function eq. (26) ($\beta = \pi/2$): $\alpha = 0.5$ (thin line), $\alpha = 1.0$ (dashed line), $\alpha = 2.0$ (thick line).

shape of the frequency spectrum in the mode decomposition of a dynamical variable x into x_ω to satisfy $\frac{d}{dt}x_\omega = i\omega x_\omega$ by proposing to treat this equation as a stochastic equation, or to regard the frequency ω as a stochastic process.

Following Kubo's prescription to replace $e^{i\omega t}$ by $\exp(i\int_0^t \omega(t')dt')$ in the random frequency modulation process, we do make the same treatment of replacing e^{iKt} by $\exp(i\int_0^t K(t')dt')$ which is regarded as a stochastic process, and which is inserted into the definition of the characteristic function $\Phi(t,s) = < e^{iKt}|x(t) = s >$.

Near the equilibrium of the 2–level dynamics ($\rho_e = \frac{1}{Z}e^{-\beta\mathcal{H}}$; $\mathcal{H} = p^2 + \frac{|L|^2}{x^2}$ for the relative coordinate only), we can write as

$$K(t) = \bar{K} + K_{\mathrm{fl}}(t), \qquad K_{\mathrm{fl}}(t) = r(t) \quad \text{and} \quad \bar{K} = \frac{2|L|^2}{x^3} - \bar{\gamma}p, \tag{23}$$

where

$$< r > = 0 \quad \text{and} \quad \bar{\gamma} = \int_0^\infty \gamma(\tau)d\tau = 2\beta\int_0^\infty < r(t)\, r(t+\tau) > d\tau,$$

implying that the process is stationary. If we further make the assumption that the process $r(t)$ is Gaussian, we can compute $\Phi(t,s)$ as follows:

$$< \exp(i\int_0^t K(t')dt')|x(t) = s> = < e^{i\bar{K}t}|x(t) = s> < \exp(i\int_0^t r(t')dt') >$$

$$= \frac{1}{(1-\frac{2it}{\beta s})^{\frac{\nu}{2}}}\exp\left[-\frac{\bar{\gamma}^2}{4\beta}t^2 - \int_0^t (t-\tau) < r\, r(\tau) > d\tau\right].$$

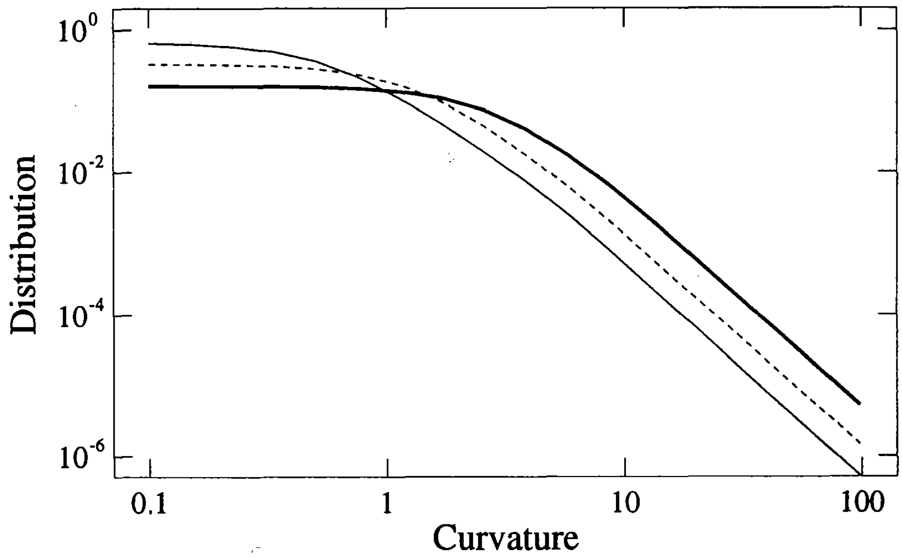

Figure 3: Curvature distribution in a logarithmic scale for GOE statistics obtained from the intermediate characteristic function eq. (26) ($\beta = \pi/2$): $\alpha = 0.5$ (thin line), $\alpha = 1.0$ (dashed line), $\alpha = 2.0$ (thick line). Each straight part corresponds to the K^{-3} line.

We further follow Kubo's ansatz of an exponential decay of the auto correlation function of $r(t)$ in the exponential, i.e.

$$< r\ r(\tau) > = \frac{1}{2} < R_1 R_1(\tau) > = \frac{\Delta^2}{2} e^{-\tau/\tau_c} \quad \text{hence} \quad \bar{\gamma} = \beta \Delta^2 \tau_c \tag{24}$$

which leads us to an expression

$$\Phi(t,s) = \frac{1}{(1 - \frac{2it}{\beta s})^{\frac{\nu}{2}}} \exp\left[-\frac{\beta \Delta^4}{4} \tau_c^2 t^2 - \frac{\Delta^2}{c} \tau_c \{ t - \tau_c (1 - e^{-t/\tau_c}) \} \right].$$

An inspection of this formula shows that by setting

$$\Delta = \frac{1}{s} \quad \text{and} \quad \Delta \tau_c = \alpha \qquad \text{hence} \quad \tau_c = \alpha s, \tag{25}$$

the function $\Phi(t, s)$ is indeed a single function $\phi(t/s)$ with the two dimensionless parameters β and α:

$$\Phi(t,s) = \phi(\frac{t}{s}) = \frac{1}{(1 - \frac{2it}{\beta s})^{\frac{\nu}{2}}} \exp\left[-\frac{\beta \alpha^2}{4} (\frac{t}{s})^2 - \frac{\alpha}{2} \{ \frac{t}{s} - \alpha (1 - e^{-t/\alpha s}) \} \right]. \tag{26}$$

Here the parameter α represents the relative correlation length in accord with Kubo's parameter to measure the degree of broadening: In the limit $\alpha \to 0$ (motional narrowing limit), $\phi(t)$ reduces to (13) with t replaced by $2t/\beta$, for which the expression (1) results with the scaling constant $B_\nu = \frac{1}{2}\beta\sigma_\nu$. In the opposite situation $\alpha^2 \gg \beta^{-1}$ (Gaussian limit), $\phi(t)$ reduces to (14) with t^2 replaced by $\frac{\beta}{2}\alpha^2 t^2$, for which the expression (2) results with $B'_\nu = \frac{\sigma_\nu}{\alpha}\sqrt{\frac{2}{\beta}}$. Figs. 2, 3 show our numerical studies about the intermediate situation between them.

Thus, our understanding of the nonuniversal feature disclosed in the previous numerical experiments on the parametric motion of levels for different systems is the variant degree of the correlation length of an avoided crossing of the individual pair, which we now believe to be true: The short correlation (rapid modulation) in Case (A) and the long correlation (slow modulation) in Case (B) indeed account for the two pictures illustrated respectively in Fig. 1(c) and (d).

Acknowledgement

Useful communications with Dr. J. Zakrzewski are gratefully acknowledged.

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Appendix　曲率分布積分公式の導出

$$P_{curv}(K) = \int_{-\infty}^{\infty} F(K,s)P_{sp}(s)ds \tag{A}$$

Part II Fig. 1(a)、(b) にみられるようなパラメーター λ の準位図において、1 対の準位 $(x_1(\lambda), x_2(\lambda))$ をえらぶとき、これが以下の条件すなわち、

$$\boxed{x_1、x_2 \text{ は最隣接準位であり、} x_2(\lambda) - x_1(\lambda) = s} \tag{A1}$$

に従う場合の、(1 対の準位 x_1、x_2 に関して定義される) 曲率 K

$$K(= K(\lambda)) = \frac{d^2}{d\lambda^2}\frac{1}{2}(x_2(\lambda) - x_1(\lambda)) \tag{A2}$$

の条件付き確率分布密度を $F(K,s)$ と書く。N–準位の結合確率分布密度を $P(\{x_n\})$ $(= P(x_1, x_2, \ldots, x_N))$ とするならば、1 対の準位 x_1、x_2 に関する曲率分布は次の積分によって計算される:

$$P_{curv}(K) = \int\cdots\int F(K, x_2 - x_1)P(x_1, x_2, \ldots, x_N)dx_1 dx_2 \cdots dx_N \tag{A3}$$

(積分の範囲は $P(\{x_n\})$ の定義される $\mathbf{R}^N$ 内の一定領域 (A4)) 以下の前提のもとで、積分 (A3) は表式 (A) に一致することが示される。

(0) 規格化条件　$$\int\cdots\int_{R_0} P(x_1, x_2, \ldots, x_N)dx_1 dx_2 \cdots dx_N = 1$$

(1) 対称性　$$\pi P(x_1, x_2, \ldots, x_N)(= P(x_{\pi 1}, x_{\pi 2}, \ldots, x_{\pi N})) = P(x_1, x_2, \ldots, x_N)$$

$$P(-x_1, -x_2, \ldots, -x_N) = P(x_1, x_2, \ldots, x_N)$$

(2) 準位反発　$$P(\ldots, x_m = x_n, \ldots) = 0$$

(3) (弱) 定常性　$$N\int\cdots\int P(x, x_2, \ldots, x_N)dx_2 \cdots dx_N = \rho \ (x \text{ によらぬ定数})$$

条件 (A1) に従う (A3) の積分はその範囲が次のように指定される:

$$P_{curv}(K) = C\int_{I(s)} dx_1 dx_2 \int_{O(s)} dx_3 \cdots dx_N F(K, x_2 - x_1)P(\{x_n\}) \tag{A4}$$

$I(s)$: 区間 $[-\frac{s}{2}, \frac{s}{2}]$ の内部　　$O(s)$: 同、外部

注 1 (0) の規格化条件のもとで K に関する規格化 $\int F(K,s)dK = 1$ は $P_{curv}(K)$ の規格化 $\int P_{curv}(K)$ $dK = 1$ を保証する。

注 2 反転対称性 (1) のもとで、一般的に $P_{curv}(-K) = P_{curv}(K)$ も成り立つ。

定数 C は、積分 (A4) における F を 1 とするときの積分が 1 に等しくなる規格化定数、すなわち

$$C = \left[\int_{I(s)} dx_1 dx_2 \int_{O(s)} dx_3 \cdots dx_N P(\{x_n\})\right]^{-1} = \frac{N(N-1)}{2\rho} \qquad \text{(A5)}$$

で与えられる。その結果、$s \geq 0$ で定義される次の積分は確率密度関数となるが、それがすなわち $P_{sp}(s)$ であり、したがって (A) が得られる。

$$C \int_{O(s)} dx_3 \cdots dx_N P(\{x_n\}) = P_{sp}(s). \qquad \text{(A6)}$$

<u>証明</u>　ランダム行列の理論で知られているように (*cf.* M.L. Mehta)、$P_{sp}(s)$ は長さ s の区間がランダム点によって占められない確率 $E(s)$ の 2 次微分で与えられる。正確に表すならば

$$P_{sp}(s) = \frac{1}{\rho}\frac{d^2E(s)}{ds^2}, \quad E(s) = \int \cdots \int_{O(s)} dx_1 dx_2 \cdots dx_N P(\{x_n\}). \qquad \text{(A7)}$$

ここで、$P(\{x_n\})$ に関する条件 (0) $\sim$ (2) のもとに

$$\frac{d^2}{ds^2}E(s) = N(N-1)\int \cdots \int_{0<s\leq x_3, x_4, \ldots, x_N} dx_3 \cdots dx_N P(-\frac{s}{2}, \frac{s}{2}, x_3, \ldots, x_N), \qquad \text{(A8)}$$

さらに条件 (3) を入れて (A8) から (A5)、(A6) が導かれる。